Using SELCIE to investigate screened scalar field models sourced by complex systems

By **C. Briddon**, C. Burrage, A. Moss, & A. Tamosiunas.

University of Nottingham

Is dark energy a scalar field?

- Currently contributes ~70% of the energy content of the universe.
- Dominates in late time, leading to an accelerating expansion rate.
- One possible explanation is a scalar field (referred to as quintessence).
- Scalar fields coupled to matter are heavily constrained by fifth force experiments.

Deriving the Chameleon (1)

- Start with a scalar field coupled to gravity and perform a conformal transformation ($\hat{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$) to get the action in the Einstein frame:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{pl}{}^2}{2}A^2(\phi)R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \; - V(\phi) + L_m\left(g_{\mu\nu}, \psi_i\right)\right)$$

$$S = \int d^4x\sqrt{-\hat{g}}\left(\frac{M_{pl}{}^2}{2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\hat{\nabla}_\mu\phi\hat{\nabla}_\nu\phi \; - \hat{V}(\phi) + \hat{L}_m\left(\hat{g}_{\mu\nu}, \psi_i\right)\right)$$

Deriving the Chameleon (2)

- Define coupling strength as: $\beta = M_{pl} \frac{A'}{A}$ → Assume constant β → $A(\phi) = e^{\beta\phi/M_{pl}}$

- The non-relativistic equation of motion of ϕ is:

$$\Box \phi = V'_{eff}(\phi)$$

$$V_{eff}(\phi) = V(\phi) + \rho A(\phi)$$

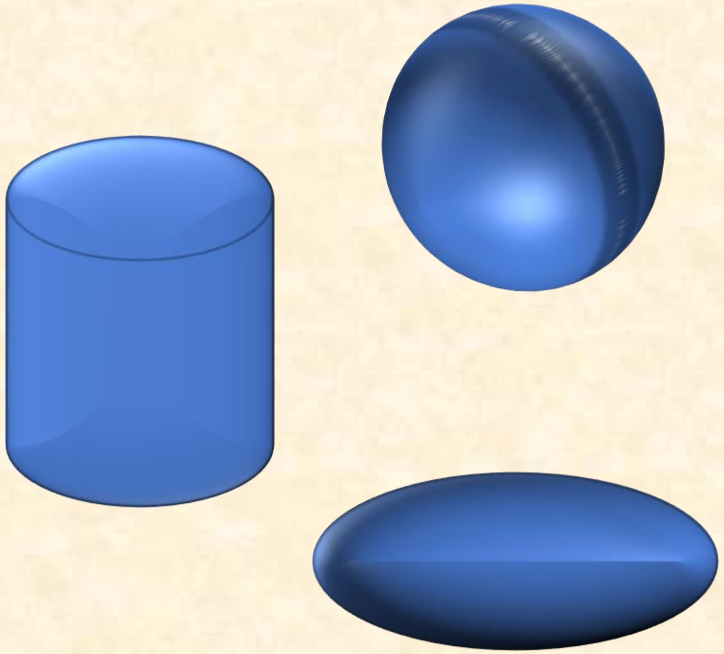
- If the potential is monotonically decreasing, the field equation will posses a minimum field value.

Chameleon Mechanism

- We will assume a field potential of the form: $$V(\phi) = \Lambda^4 \left(1 + \left(\frac{\Lambda}{\phi}\right)^n\right)$$
- The static field equation is therefore: $$\nabla^2\phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{pl}}$$
- The field value that minimises the potential is then: $$\phi_{min} = \left(\frac{n\Lambda^{n+4}M_{pl}}{\beta\rho}\right)^{1/(n+1)}$$
- The corresponding a Compton wavelength of: $$\lambda^2 = \frac{(n\Lambda^{n+4})^{1/(n+1)}}{(n+1)}\left(\frac{\beta\rho}{M_{pl}}\right)^{-\frac{n+2}{n+1}}$$
- We see as ρ increases λ deceases leading to the field being screened.

Approximate Analytic Solutions

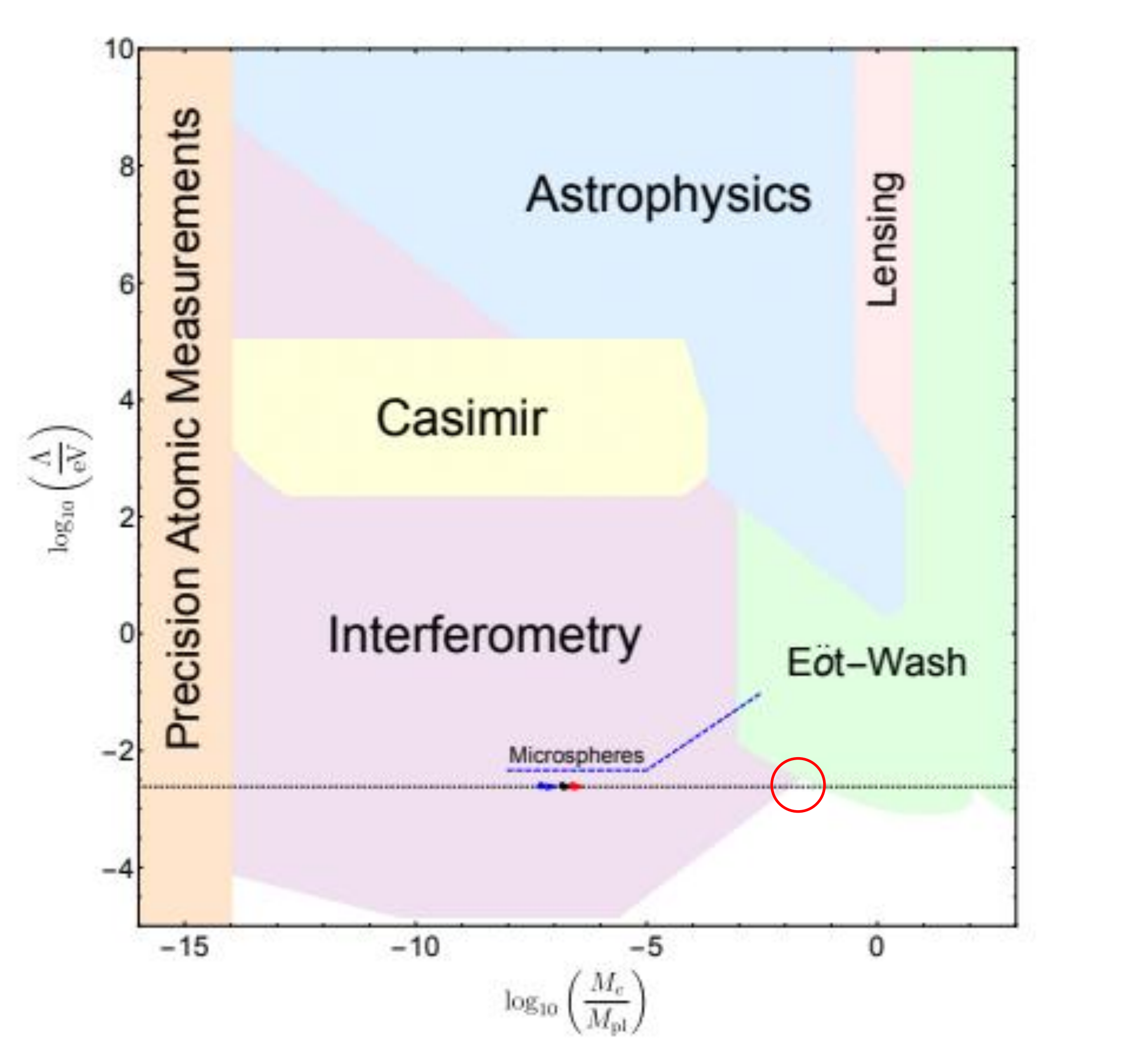
- These solutions are for highly symmetrical systems such as:
  - Spheres - $\phi(r) \approx \phi_0 - \left(\frac{3}{4\pi M}\right)\left(\frac{\Delta R}{R}\right)\frac{M_c e^{-m_0(r-R)}}{r}$
  - Cylinders - $\phi(r) \approx \phi_0 - \frac{\rho_c R^2}{2M}\left(1 - \frac{S^2}{R^2}\right) K_0(m_0 r)$
  - Ellipses - $\phi(\xi, \eta) \approx \phi_0 \left(1 - \frac{Q_0(\xi) - P_2(\eta) Q_2(\xi)}{Q_0(\xi_0)}\right)$



References: ‘arXiv:astro-ph/0309411’, ‘arXiv:1408.1409’, ‘arXiv:1412.6373’

Constraints on the model

- Chameleon model has become very constrained.
- Only a small region left where $\Lambda = \Lambda_{DE}$.
- Atom interferometry experiment used a spherical source.



[1709.09071] Tests of Chameleon Gravity (arxiv.org)

What is SELCIE?

- SELCIE (Screening Equations Linearly Constructed and Iteratively Evaluated) is a python package designed to solve the chameleon field equations for arbitrary systems.

- It does this in two parts:
  - Mesh generation using the GMSH software (http://gmsh.info/).
  - Solves the field equations using FEniCS software (http://fenicsproject.org/).

Rescaled Chameleon equation

- We rescale the values in the equation in units of their vacuum values:

$$\hat{\rho} = \rho/\rho_0 \qquad \hat{\phi} = \phi/\phi_{min}(\rho_0) \qquad \hat{\nabla} = L\nabla \qquad \alpha = \frac{M_{pl}\phi_0}{\beta L^2 \rho_0}$$

- Rescaled field equation is: $\alpha\hat{\nabla}^2\hat{\phi} = -\hat{\phi}^{-(n+1)} + \hat{\rho}$

- A nice way to interpret α is to consider its relation to the rescaled Compton wavelength:

$$\hat{\lambda}^2 = \frac{\alpha}{(n+1)}\hat{\rho}^{-\frac{(n+2)}{(n+1)}}$$

Finite Element Method (FEM)

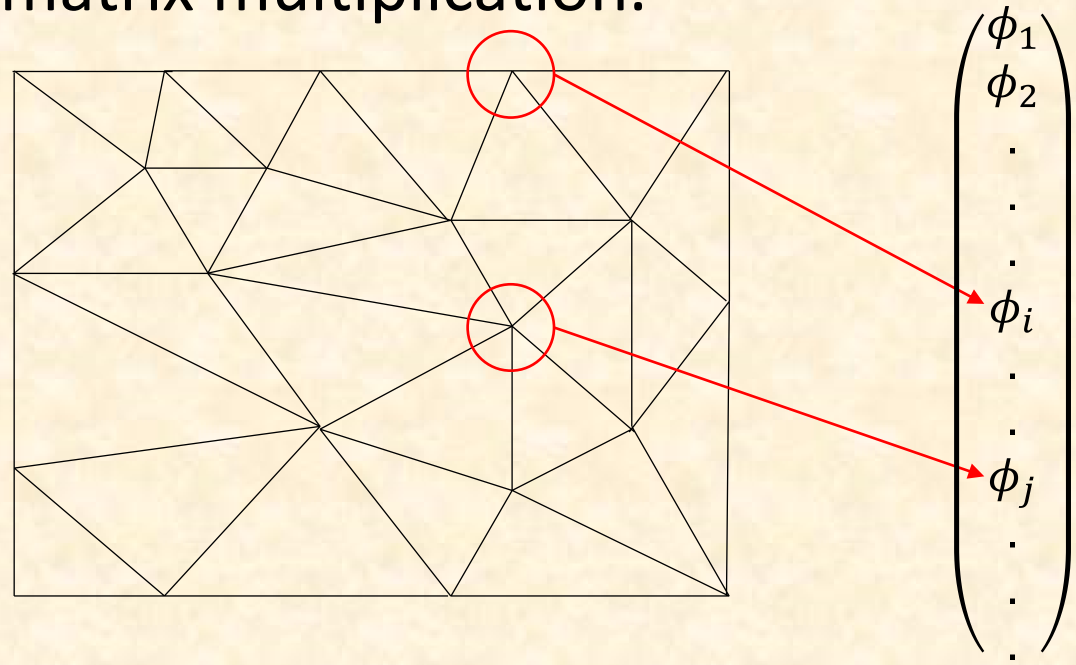
- To solve equations of the form $\nabla^2 u(x) = f(x)$, use Green's function (with zero field gradient at the boundary) and discretise the field. The problem can then described by a linear matrix multiplication.

$$\int_\Omega \nabla u \cdot \nabla v_j dx = \int_\Omega f(x) v_j dx$$

$$u(x) = \sum_i U_i e_i(x)$$

$$\sum_i \left(\int_\Omega \nabla e_i \cdot \nabla v_j dx \right) U_i = \int_\Omega f(x) v_j dx$$

$$MU = F$$



Solving the equations (Picard method)

- Expand the nonlinear part around $\hat{\phi}_k$:

$$\hat{\phi}^{-(n+1)} = \hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+2)}(\hat{\phi} - \hat{\phi}_k) + \mathcal{O}(\hat{\phi} - \hat{\phi}_k)^2$$

$$\hat{\phi}^{-(n+1)} \approx (n+2)\hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+1)}\hat{\phi}$$

- We then solve for $\hat{\phi}$ around some $\hat{\phi}_k$.

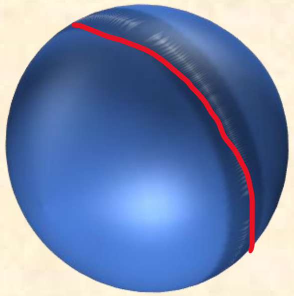
- Perform update $\hat{\phi}_{k+1} = \omega\hat{\phi} + (1-\omega)\hat{\phi}_k$.

- Repeat from first step until convergence.

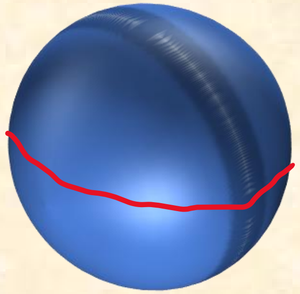
3D→2D

- When evaluating a 2D mesh we need to impose a symmetry.

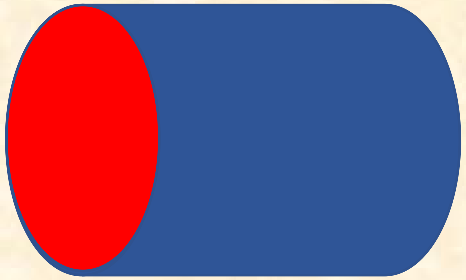
$$d^3x = \sigma d^2x$$



- Some examples include:
  - Vertical axis-symmetry - $\sigma = |x|$

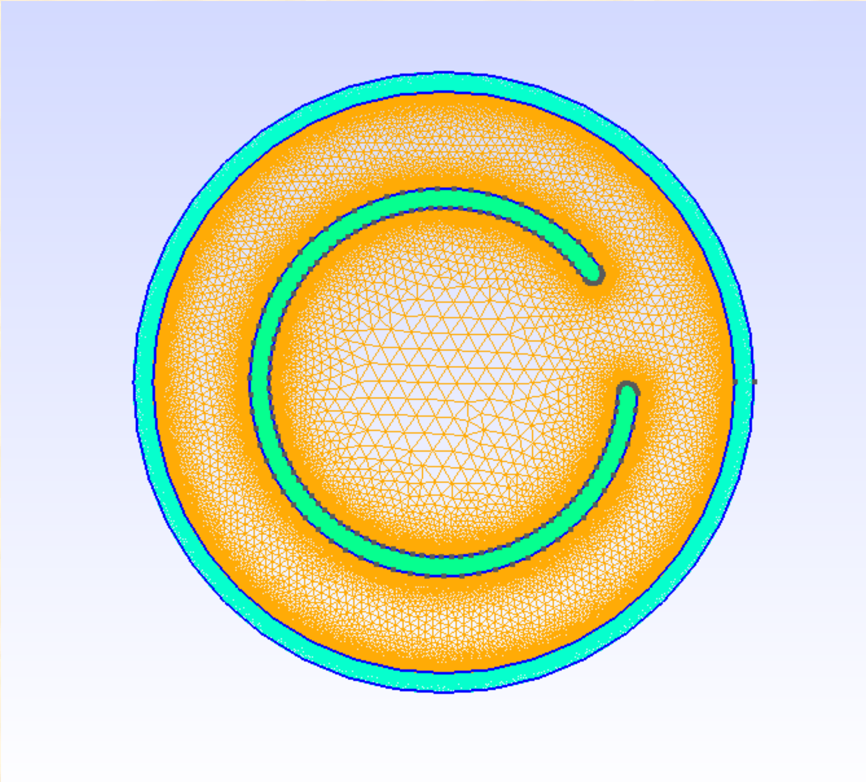
  

  - Horizontal axis-symmetry - $\sigma = |y|$
  - Cylindrical - $\sigma = 1$



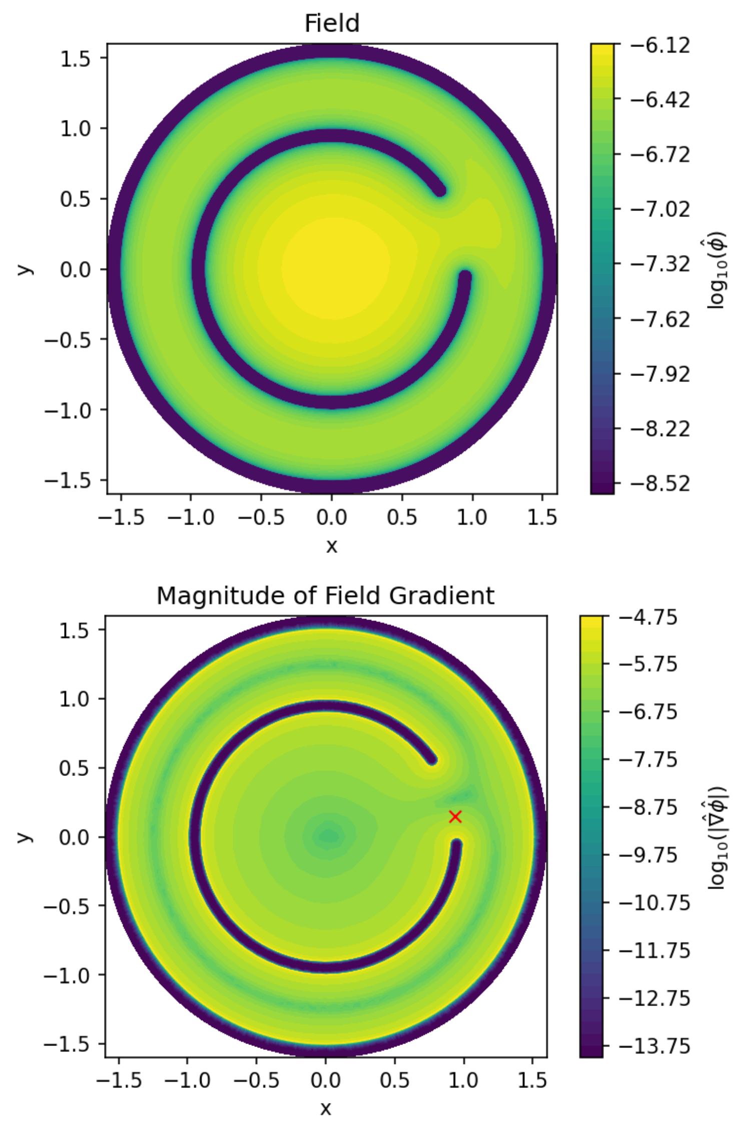
Example Code – Mesh Generating

```
MT = MeshingTools(dimension=2)

# Construct source.

MT.points_to_surface(horseshoe())

MT.create_subdomain(CellSizeMin=5e-4, CellSizeMax=0.1,
DistMax=0.5)

# Place source in vacuum chamber.

MT.create_background_mesh(CellSizeMin=1e-3, CellSizeMax=0.1,
            DistMax=0.5, background_radius=1.5, wall_thickness=0.1)

# Make mesh.

MT.generate_mesh(filename= "horseshoe", show_mesh=True)

MT.msh_2_xdmf(filename = "horseshoe")
```



Example Code – Solving the Field Equation

```
# Set parameters.
n = 1
alpha = 1.0e18
# Define density profile.
p = DensityProfile(filename=”horseshoe", dimension=2,
            symmetry='cylinder slice’, profiles=[source, vacuum,  wall])
# Solve for the gradient of the field.
s = FieldSolver(alpha, n, density_profile=p)
s.picard()
s.calc_field_grad_mag()
print(s.field_grad_mag(X[0], X[1]))  # 8.55e-07.
# Plot results.
s.plot_results(field_scale='log', grad_scale='log')
plt.plot(X[0], X[1], 'rx')
```



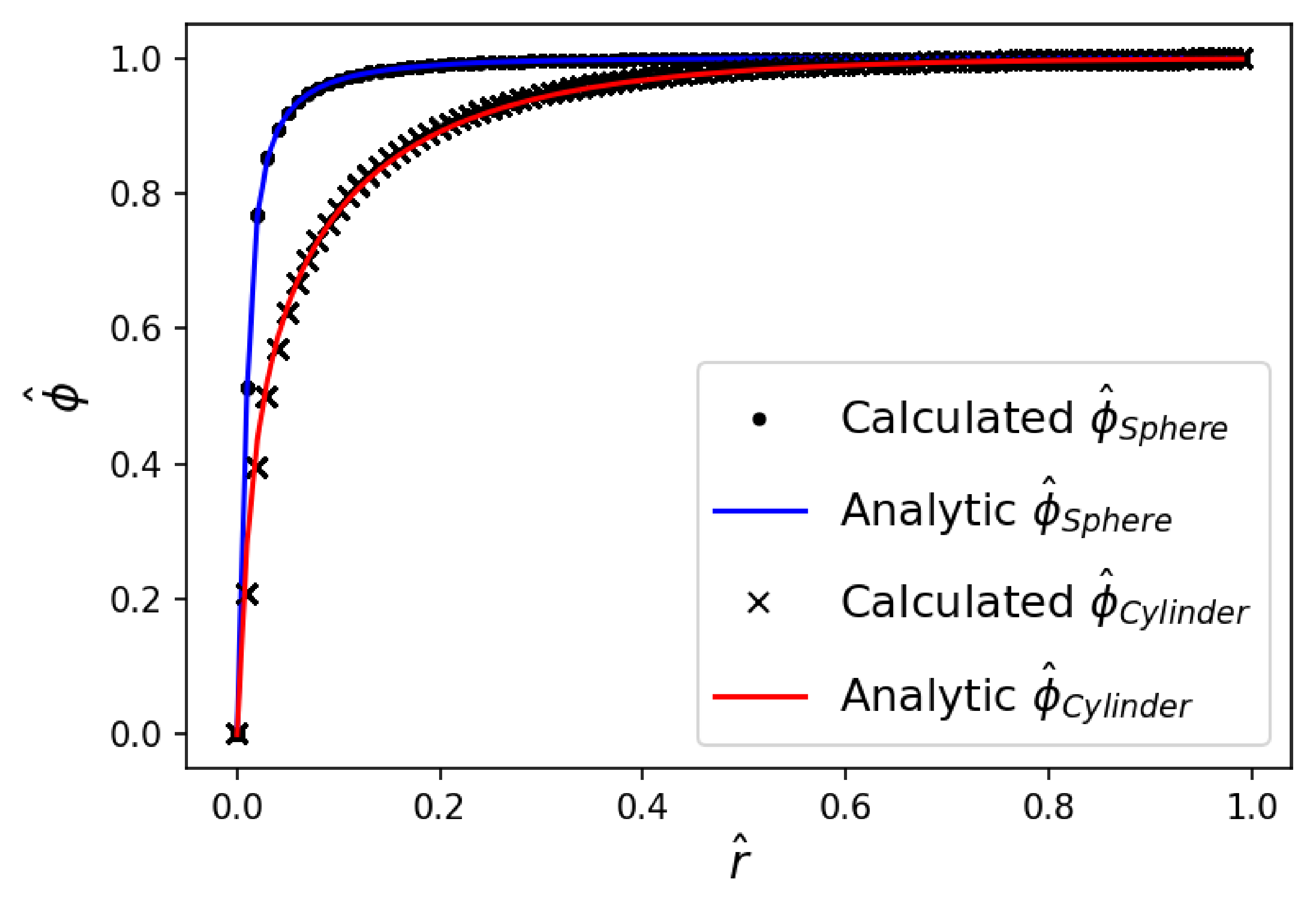
Test – Sphere & Cylinder

Sphere

$$\hat{\phi}(\hat{r}) \approx 1 - \frac{\hat{R}}{\hat{r}} e^{-(\hat{r}-\hat{R})\sqrt{\frac{(n+1)}{\alpha}}}$$

Cylinder

$$\hat{\phi}(\hat{r}) \approx 1 - \frac{\mathcal{K}_0\left(\hat{r}\sqrt{\frac{(n+1)}{\alpha}}\right)}{ln\left(\frac{4\alpha}{(n+1)\hat{R}}\right)}$$

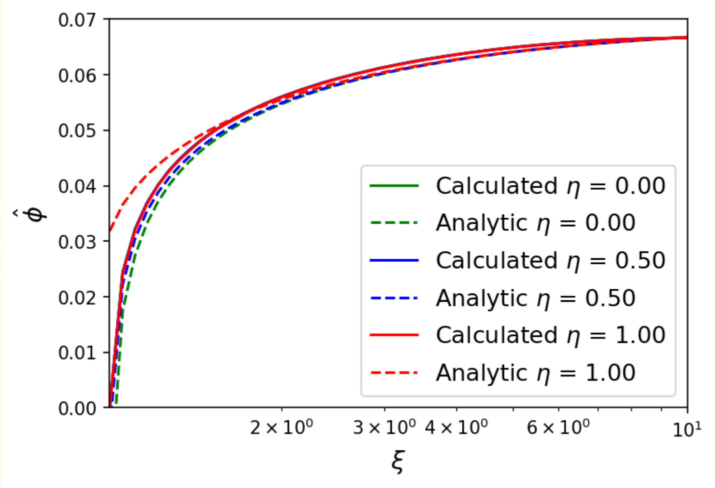


Test – Ellipse

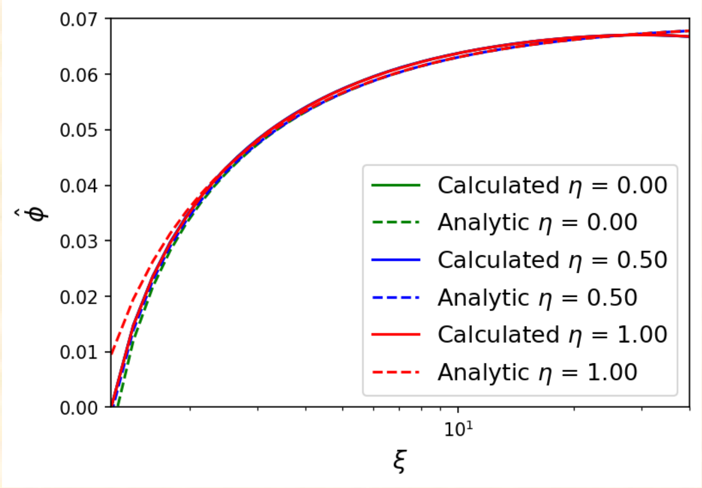
Ellipsoid

$$\hat{\phi}(\xi,\eta) \approx \hat{\phi}_0\left(1 - \frac{Q_0(\xi) - P_2(\eta)Q_2(\xi)}{Q_0(\xi_0)}\right)$$

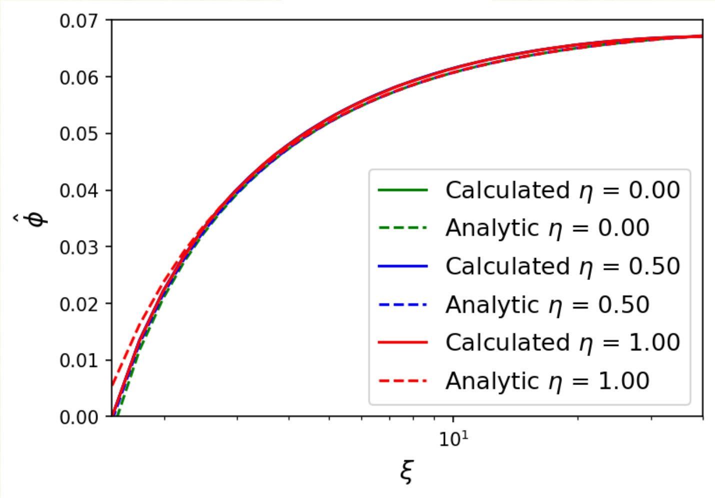
$\xi_0 = 1.01$



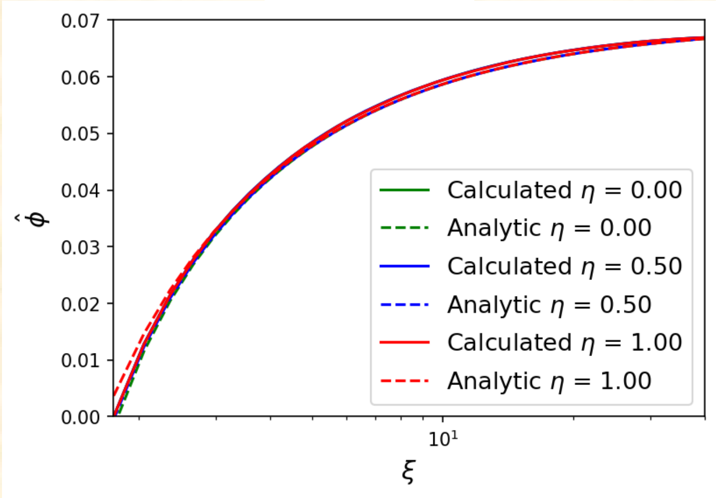
$\xi_0 = 1.25$



$\xi_0 = 1.50$

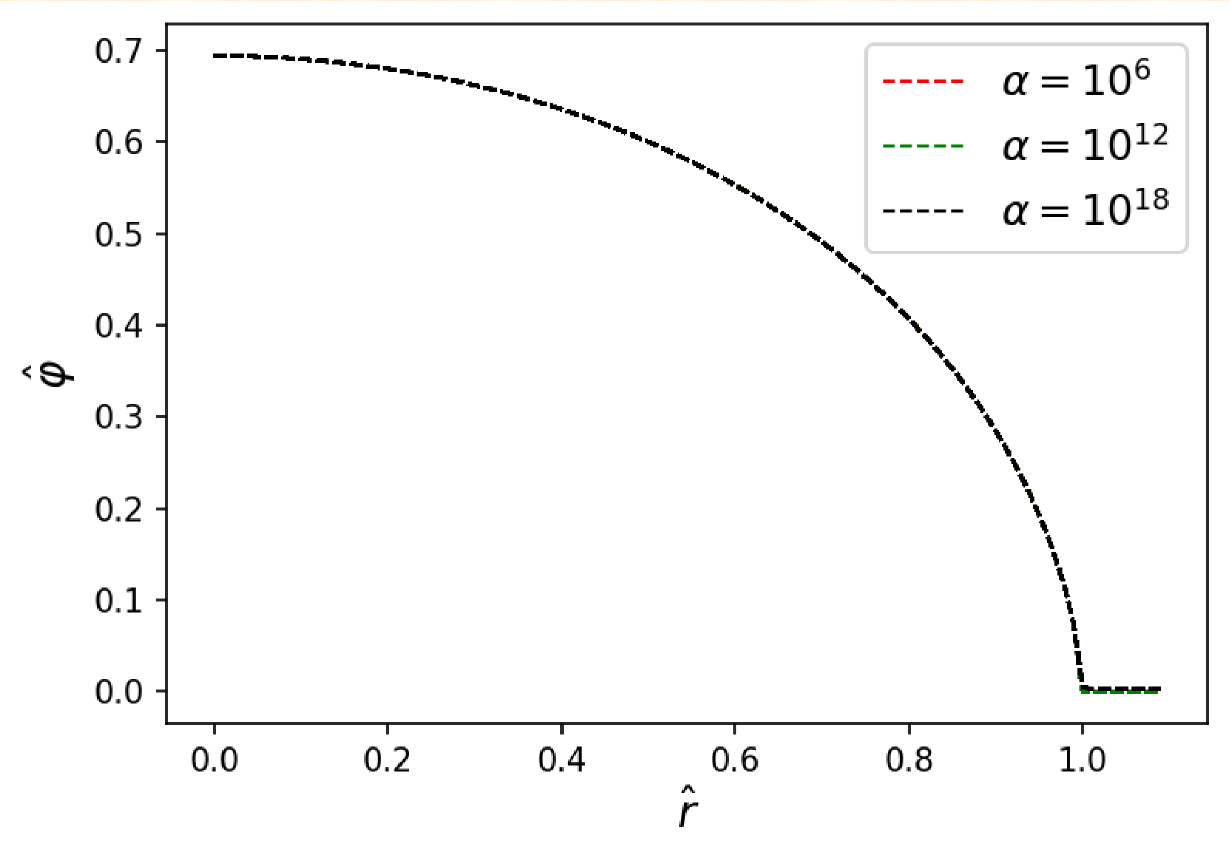


$\xi_0 = 1.75$



Test – Empty vacuum chamber

- For large α the field will not reach its maximum inside an empty chamber. Recall: $\hat{\lambda}_0{}^2 = \dfrac{\alpha}{(n+1)}$

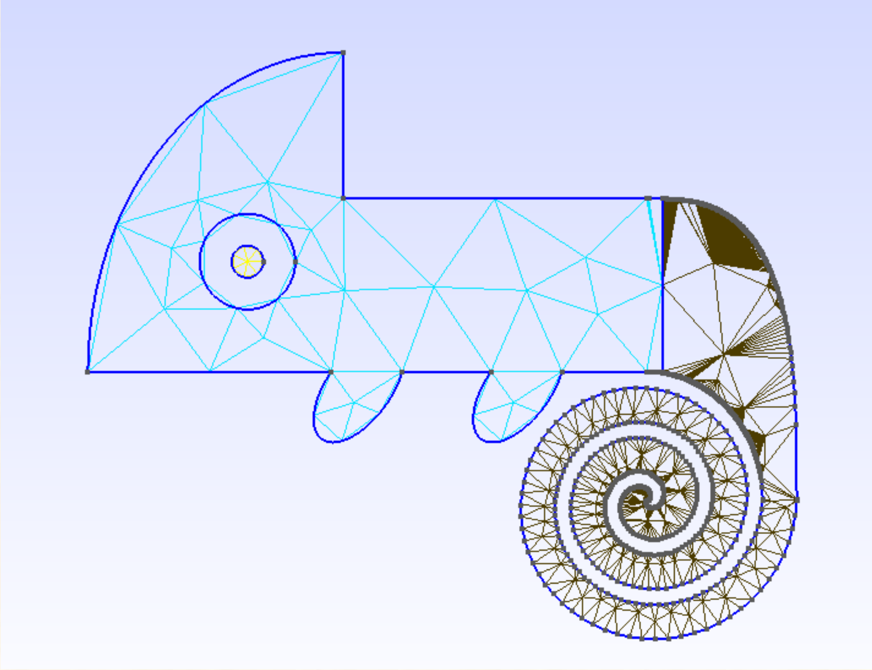


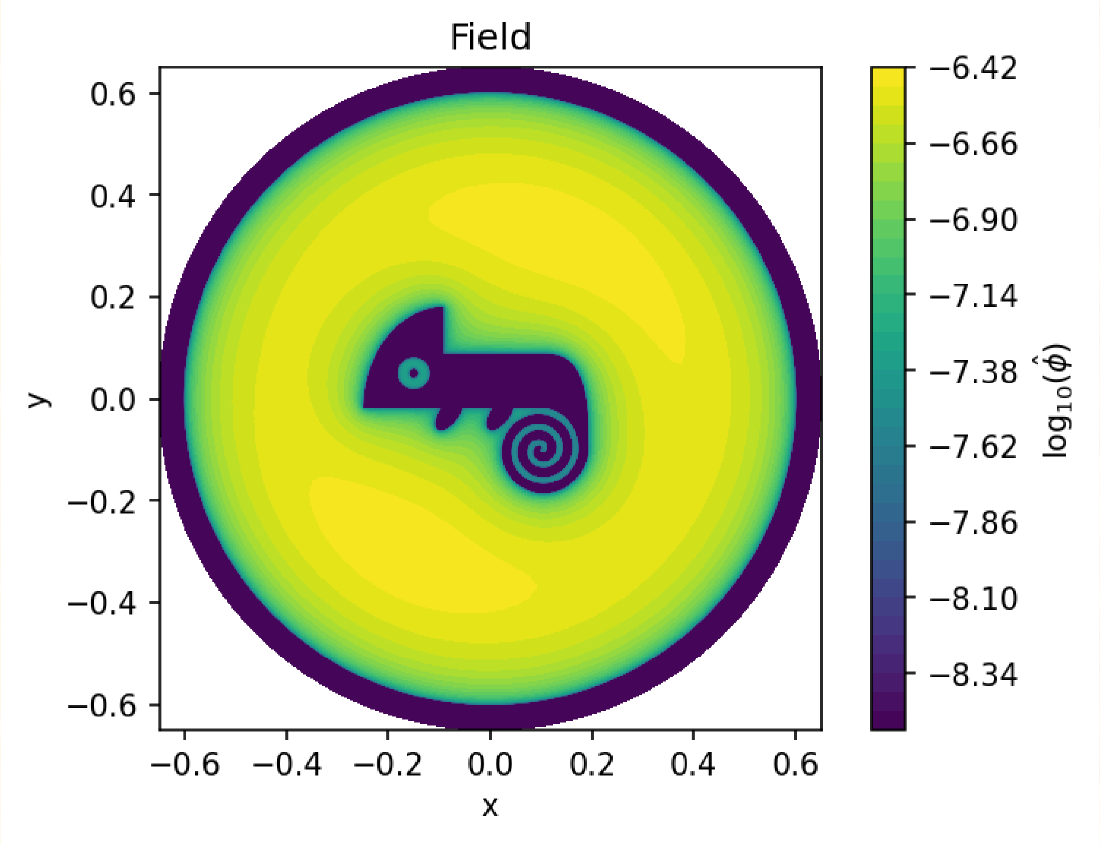
- Therefore, $\hat{V}_{eff} \approx \hat{\phi}^{-(n+1)}$.

- E.O.M. is independent of α for:

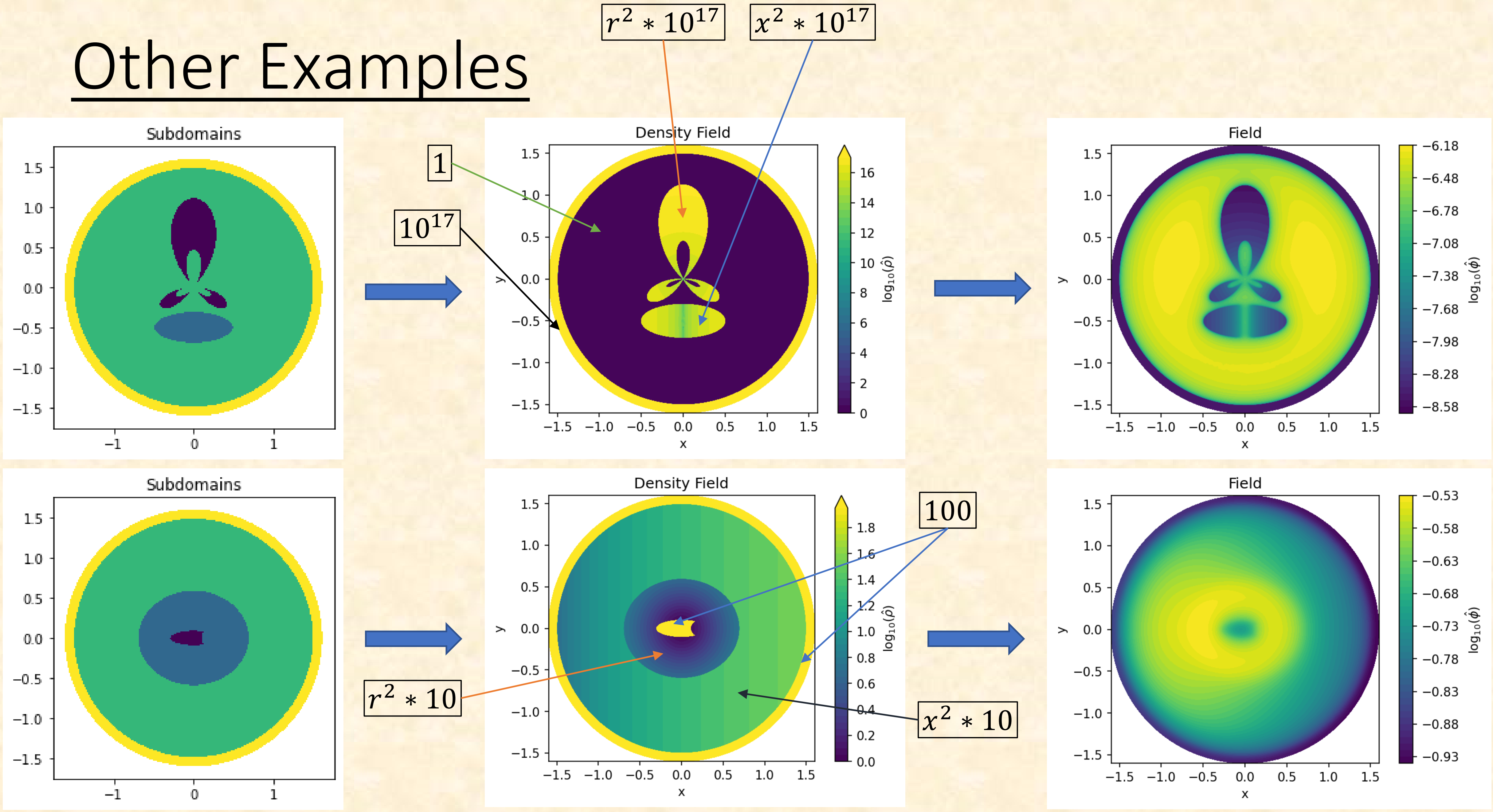
$$\hat{\varphi} = \alpha^{1/(n+2)}\hat{\phi}$$

The chameleon of a chameleon





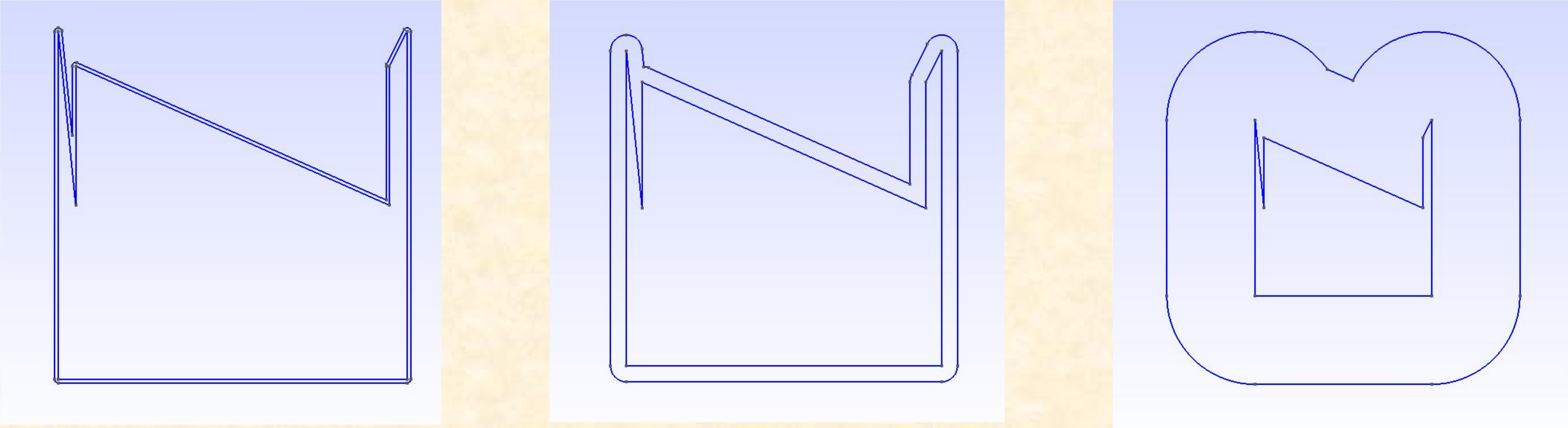
Other Examples



Measuring the fifth force

- We are interested at the fifth force a distance d from the source.

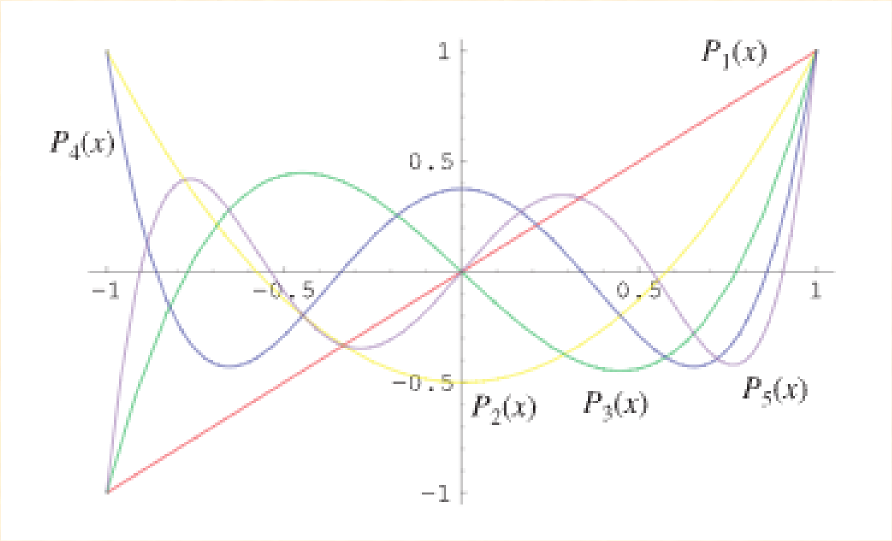
- We therefore define a boundary where to measure.



Legendre Polynomial Basis

- Legendre polynomials are solutions to:

$$(1-x^2)P_n''(x) - 2P_n'(x) + n(n+1)P_n(x) = 0$$



Legendre Polynomial -- from Wolfram MathWorld

- Examples include:
  - $P_0(x) = 1$
  - $P_1(x) = x$
  - $P_2(x) = \frac{1}{2}(3x^2 - 1)$

- Forms a basis between $(-1, 1)$:

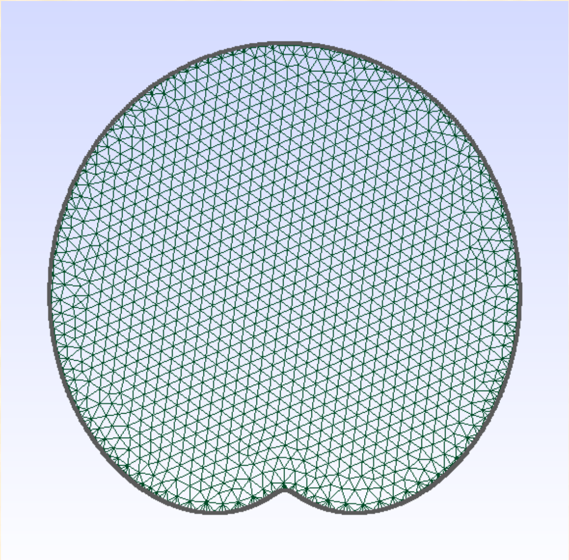
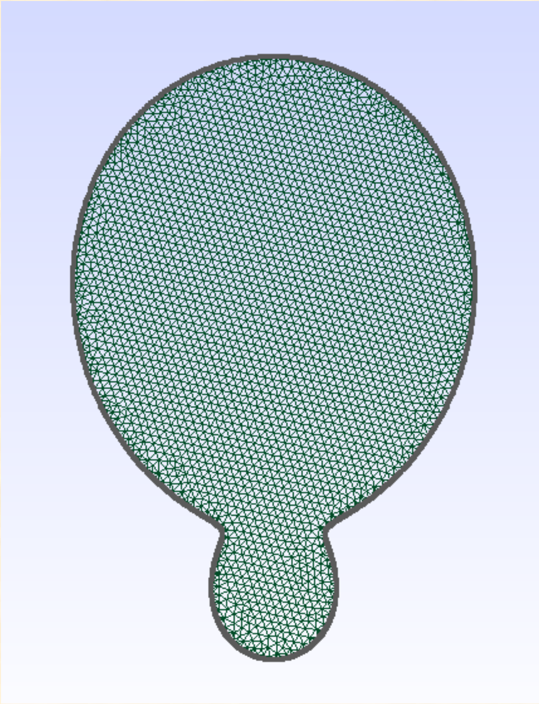
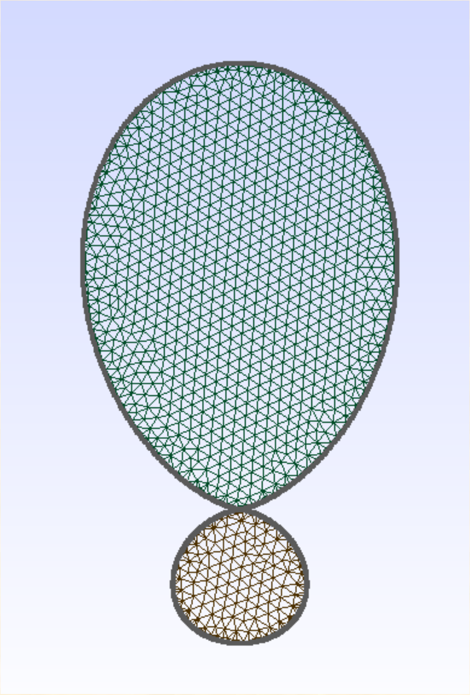
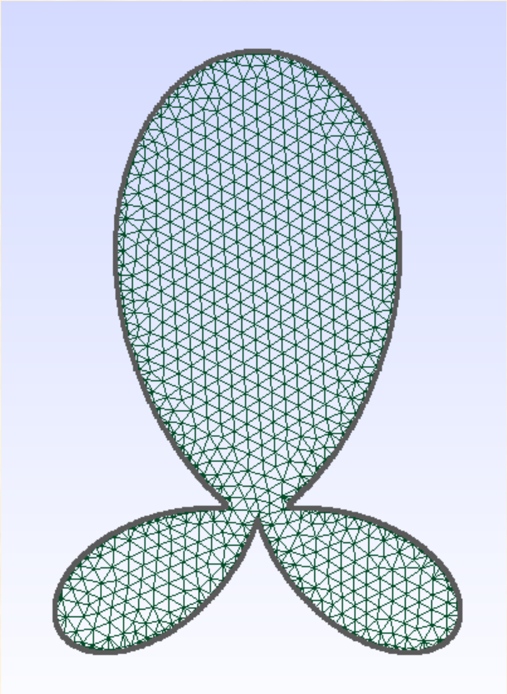
$$\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{1}{2n+1}\delta_{nm}$$

Legendre Polynomial shapes

- We used shapes defined by:

$$R(\theta) = \sum_{n=0}^{N} a_n P_n(\cos(\theta))$$

- Some examples:

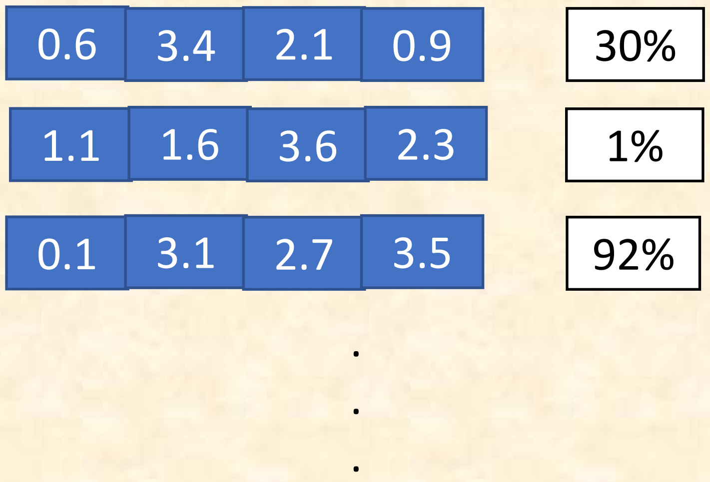





Genetic Algorithm

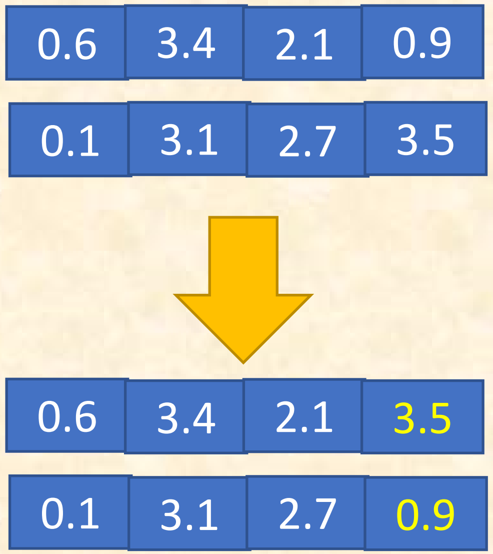
- A minimising/maximising algorithm based on organic evolution.

- Consists of 3 part:

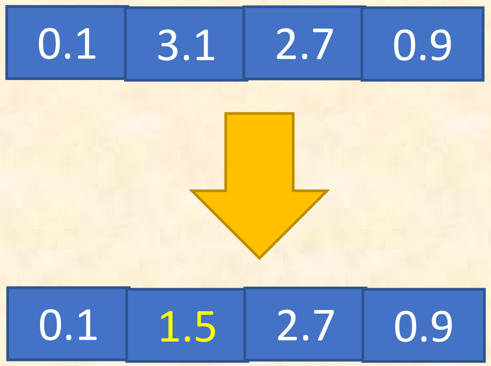
Selection



Crossover

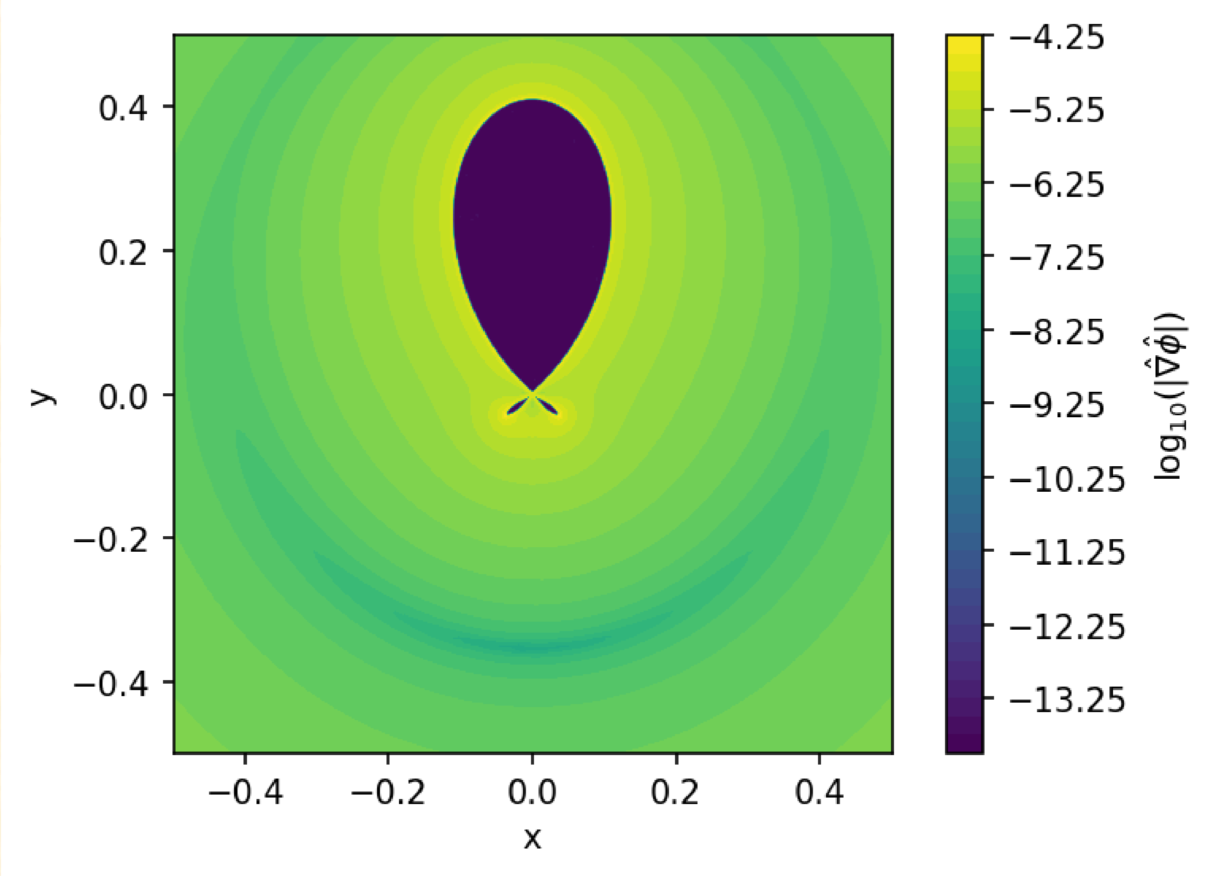


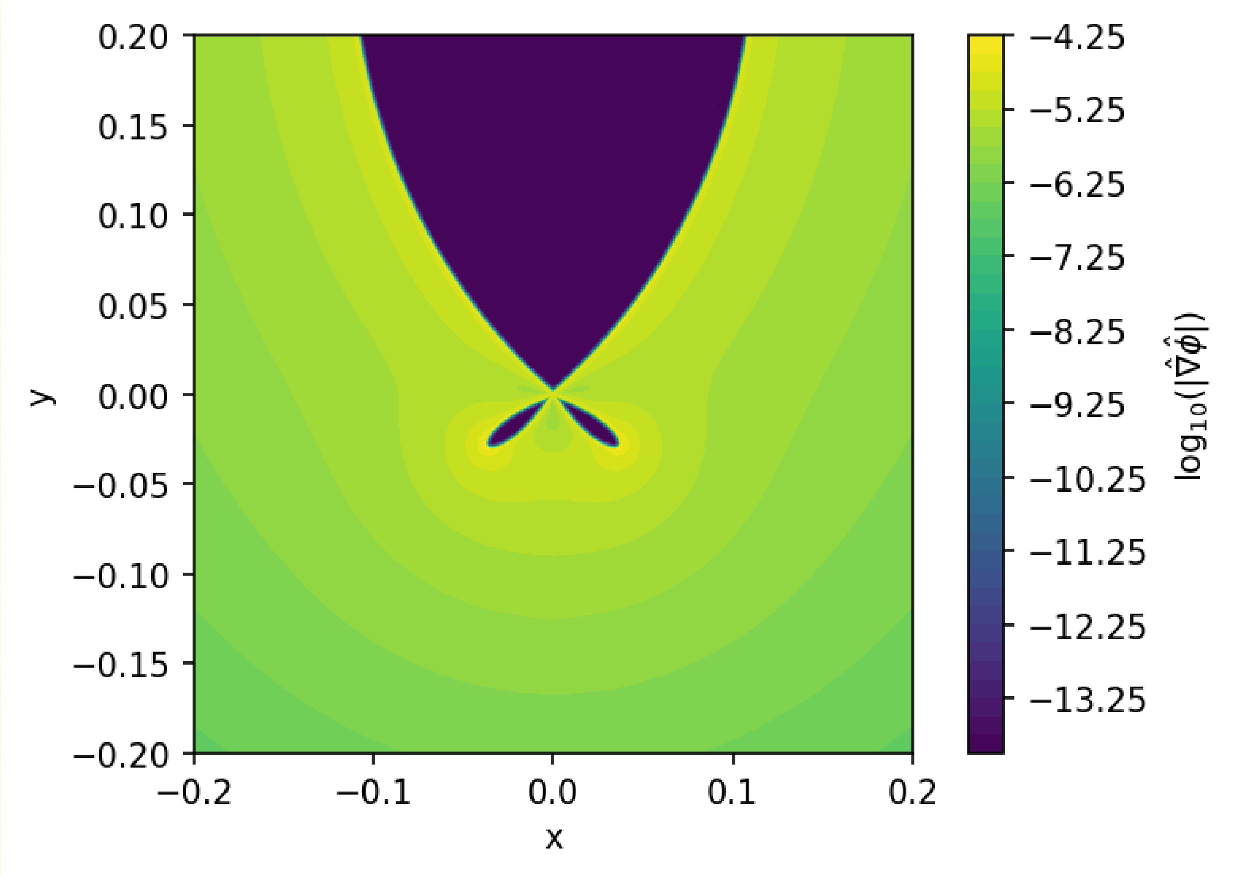
Mutation



Best shape (Nc = 4, V=0.01)

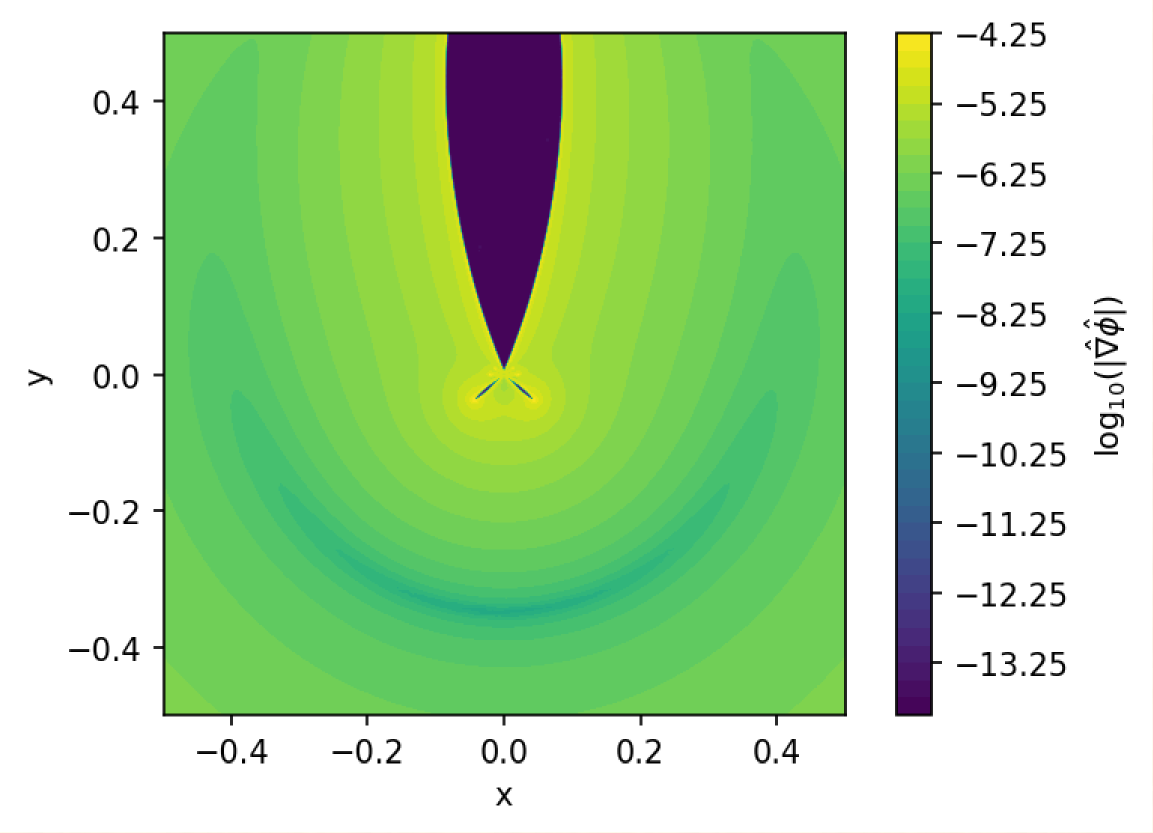
$$\hat{F} = 4.64e - 6$$

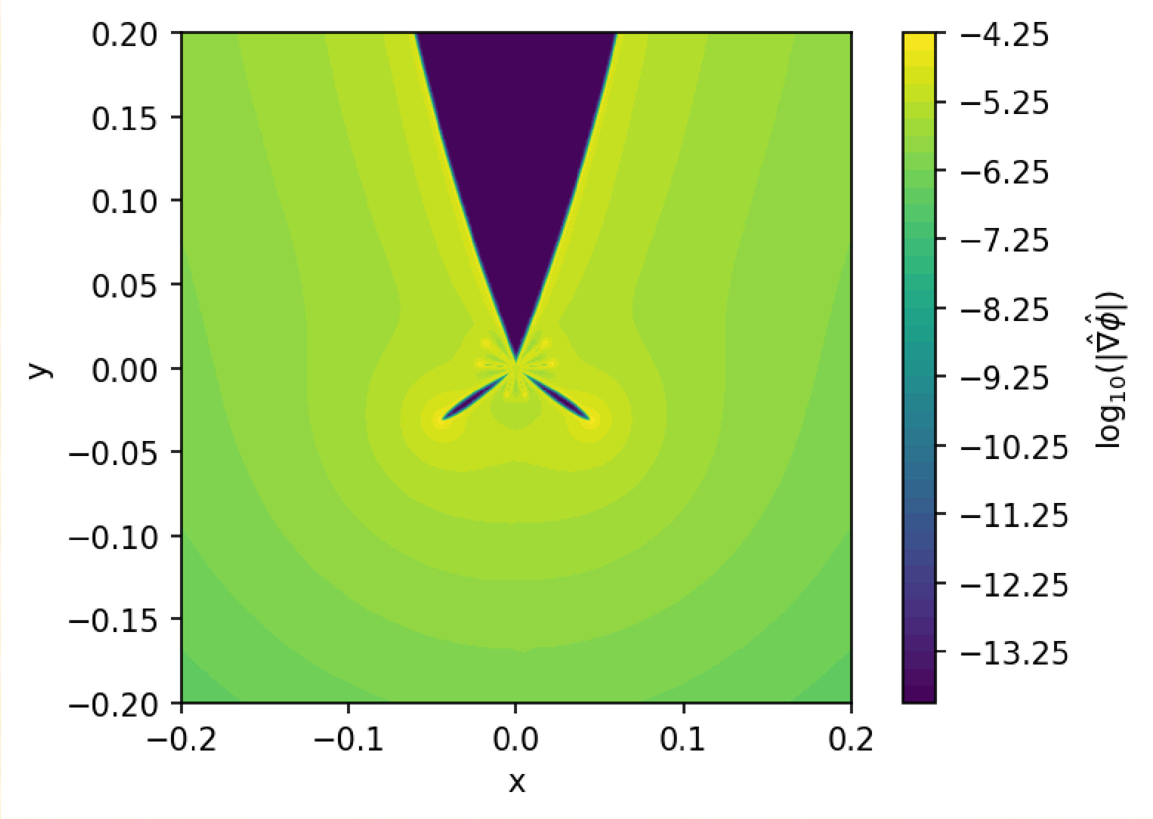




Best shape (Nc = 10, V=0.01)

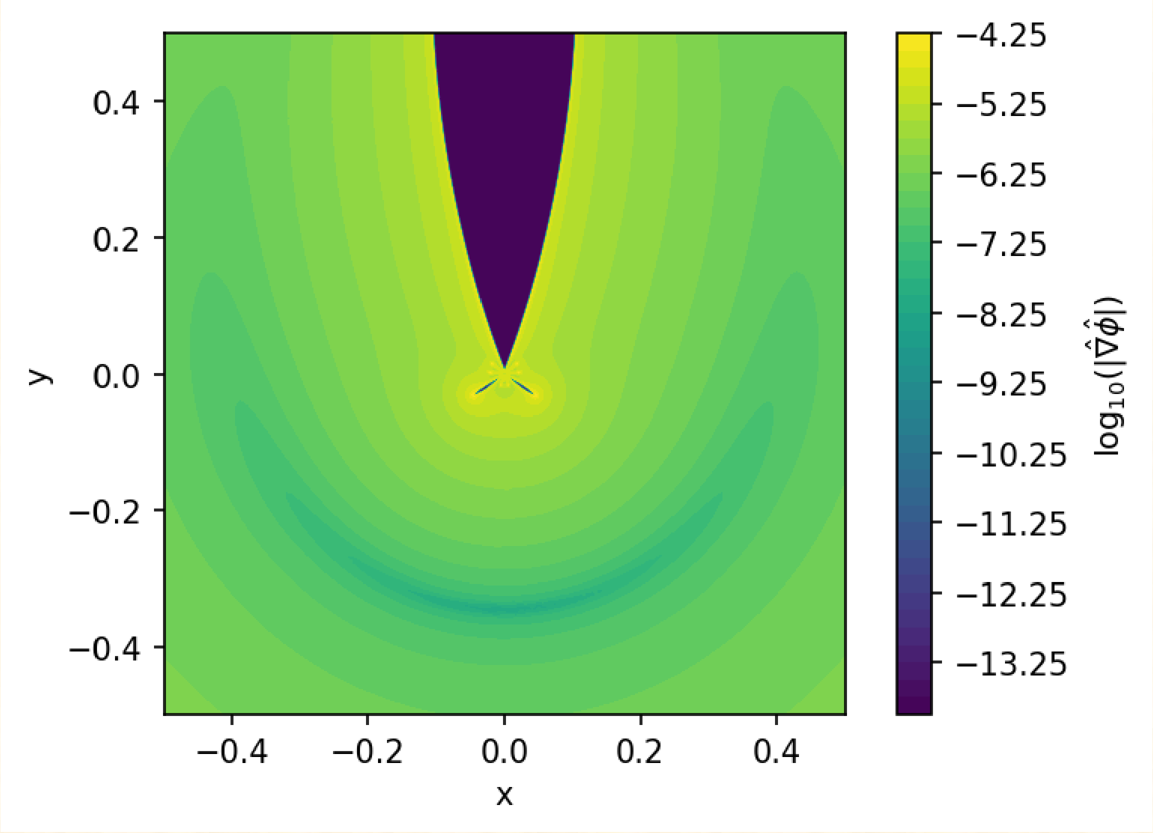
$$\hat{\mathrm{F}} = 4.95\,e-6$$

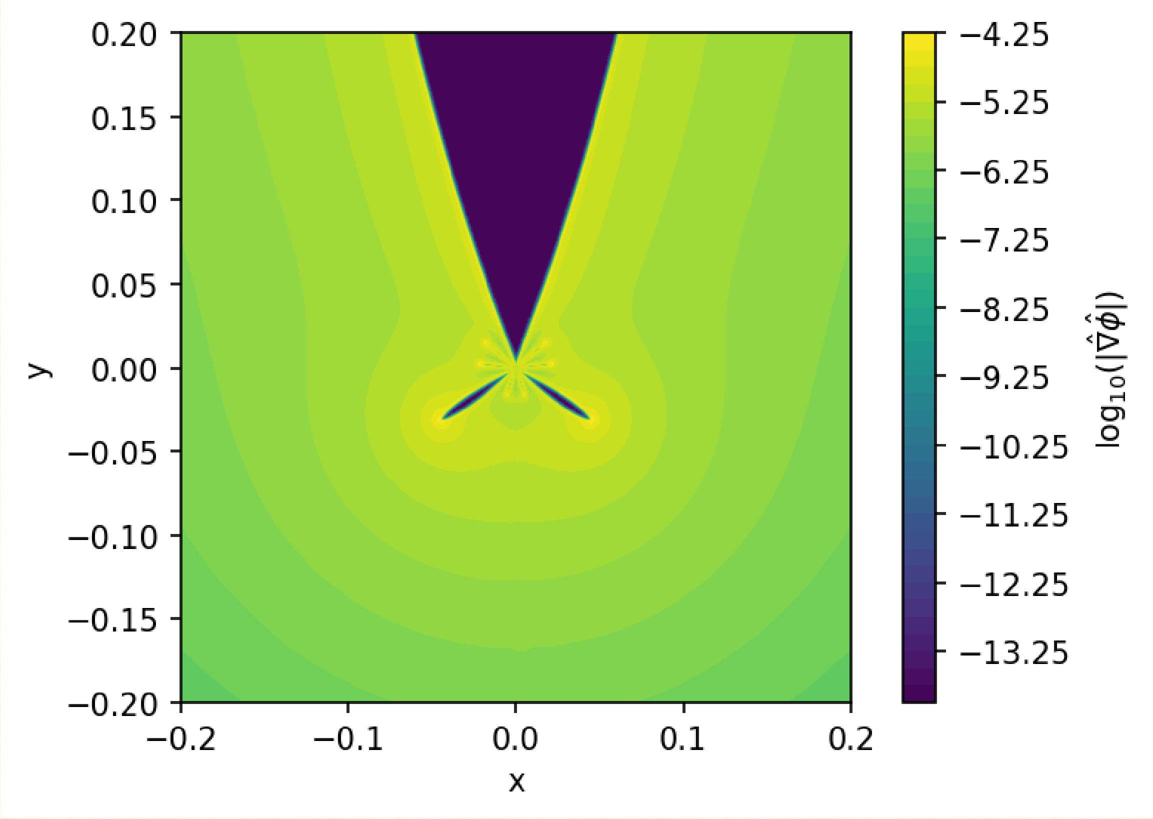


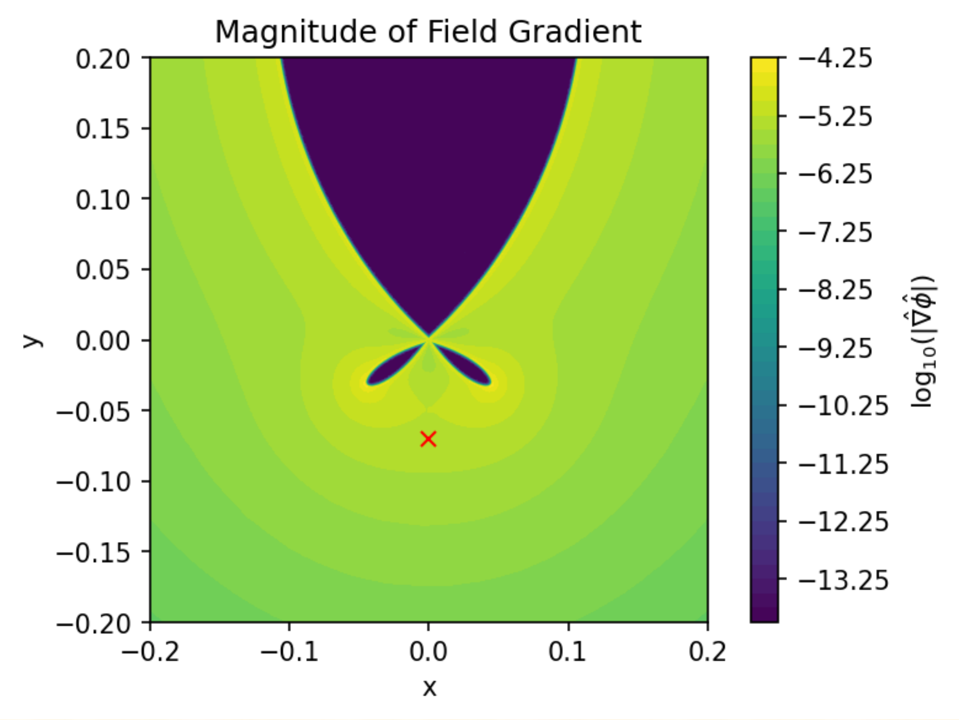


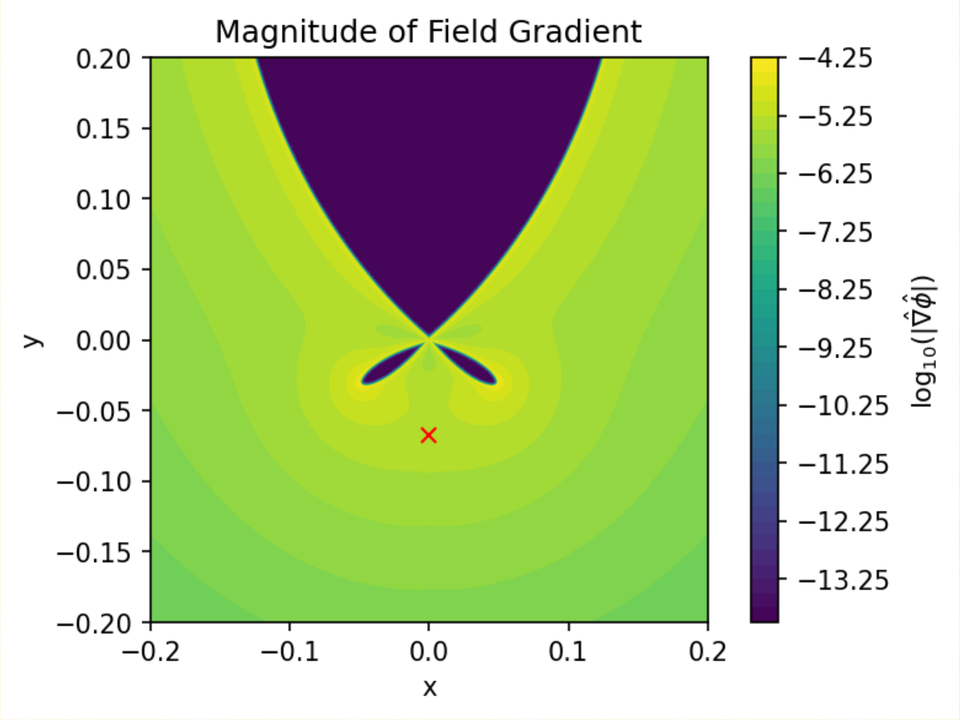
Best shape (Nc = 10, V=0.02)

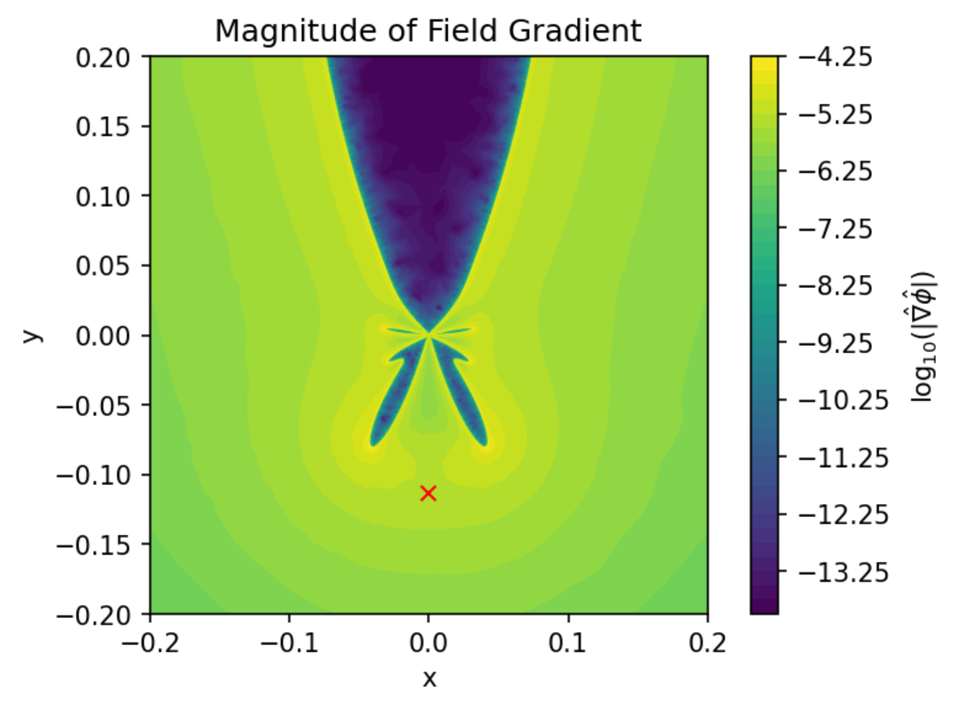
$\hat{F} = 4.92\, e - 6$

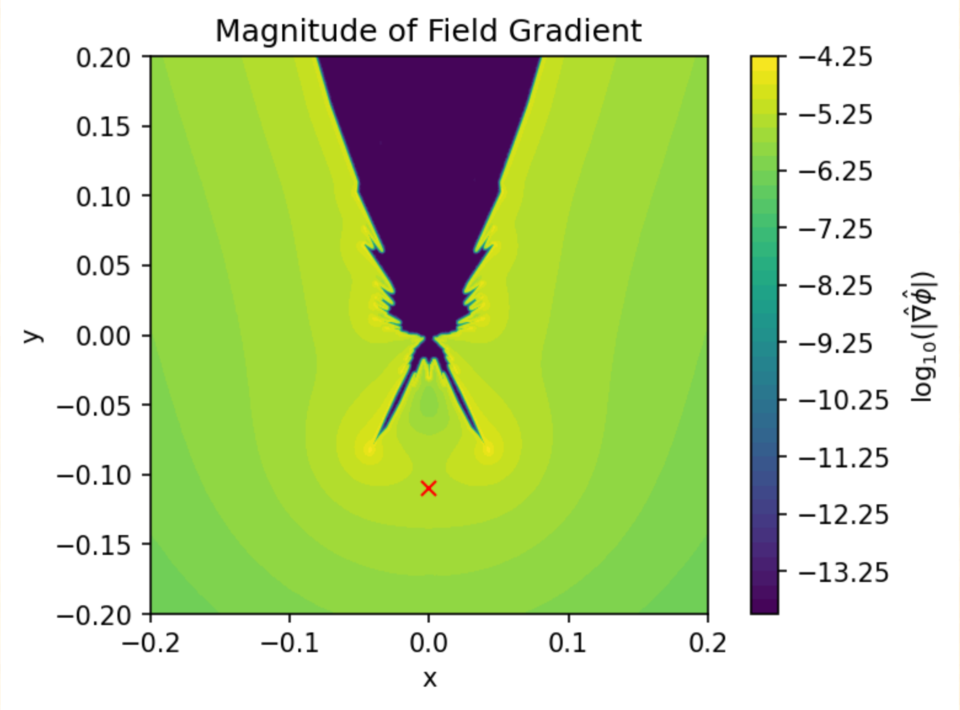




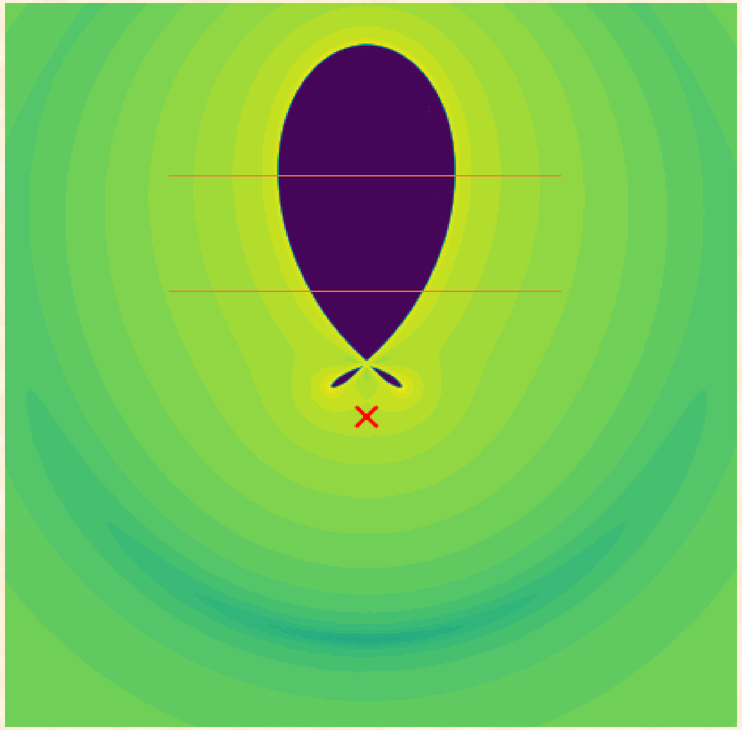

Magnitude of Field Gradient
y
x
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.2
−0.1
0.0
0.1
0.2
−4.25
−5.25
−6.25
−7.25
−8.25
−9.25
−10.25
−11.25
−12.25
−13.25
log10(|∇̂φ̂|)

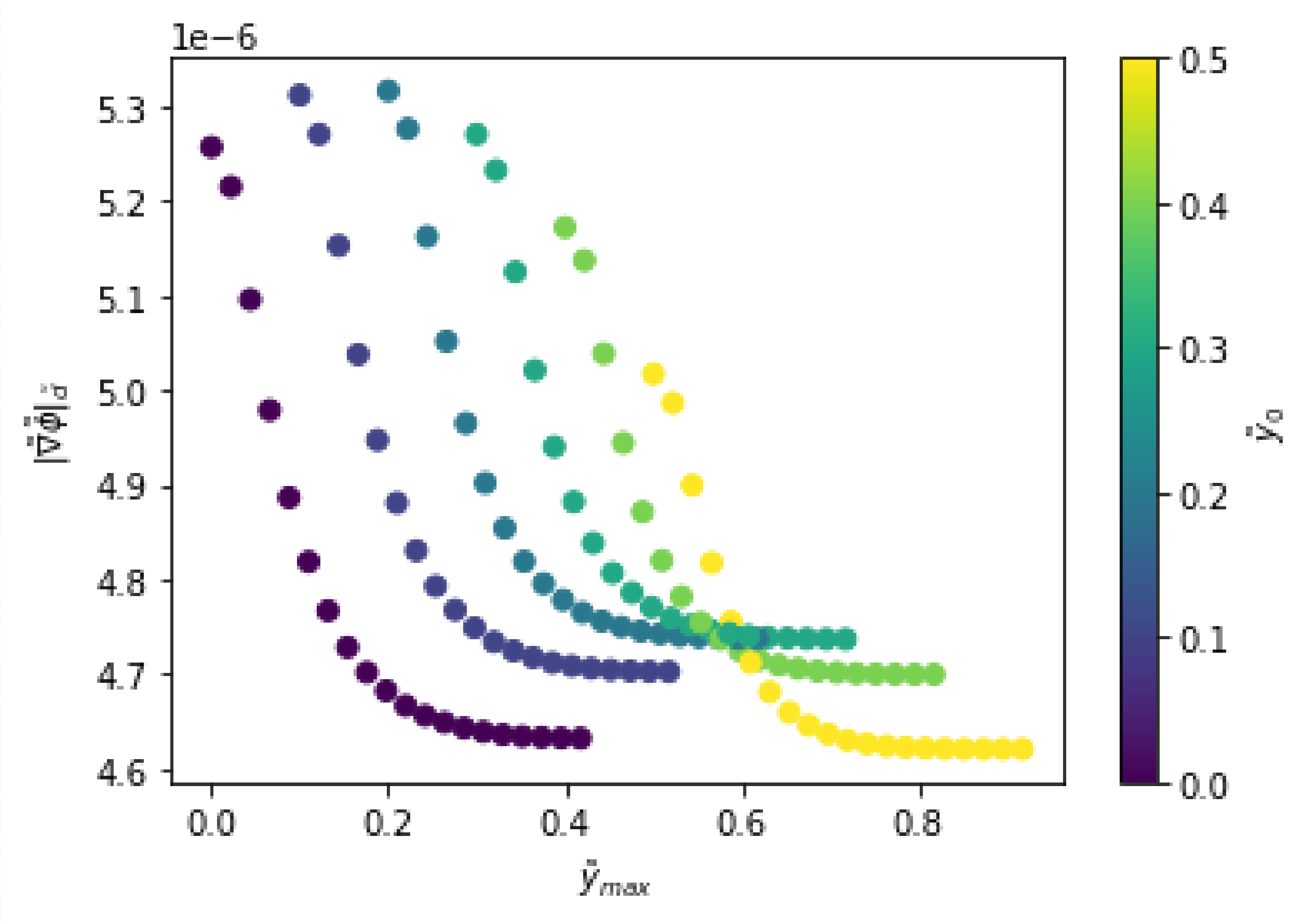

Magnitude of Field Gradient
y
x
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.2
−0.1
0.0
0.1
0.2
−4.25
−5.25
−6.25
−7.25
−8.25
−9.25
−10.25
−11.25
−12.25
−13.25
log10(|∇̂φ̂|)


Magnitude of Field Gradient
y
x
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.2
−0.1
0.0
0.1
0.2
−4.25
−5.25
−6.25
−7.25
−8.25
−9.25
−10.25
−11.25
−12.25
−13.25
log10(|∇̂φ̂|)


Magnitude of Field Gradient
y
x
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.2
−0.1
0.0
0.1
0.2
−4.25
−5.25
−6.25
−7.25
−8.25
−9.25
−10.25
−11.25
−12.25
−13.25
log10(|∇̂φ̂|)

Cutting off mass





Current/Future works

- Investigate general trends between classes of shapes.
- Introduce Neumann boundary conditions
- Developing a symmetron version (possibility of making methodology work for other models).
- Working to add time-dependence/dynamic meshes.

Thank you for listening

ArXiv: arXiv:2110.11917, arXiv:2206.06480 , arXiv:2108.10364
Github: GitHub - C-Briddon/SELCIE